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THEORY OF DUAL-CHARGED PARTICLES

Samuel Kaaloa Anderson

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THESIS

THEORY OF DUAL-CHARGED PARTICLES

by

Samuel Kaaloa Anderson

June 1977

Thesis Advisor William B. Zeleny

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In studying the interaction of dual-charged particles (particles with both electric and magnetic charge), the introduction of a two-dimensional charge space results in a set of dynamical equations governing the behavior of the system that are invariant under rotation in charge space. Systems in which all particles have the same magnetic to electric charge ratio are indistinguishable from conventional electro-dynamic systems

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Theory of Dual-Charged Particles

by

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Lieutenant, United States Navy
B.S., University of Washington, 1969

Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

In studying the interaction of dual-charged particles (particles with both electric and magnetic charge), the introduction of a two-dimensional charge space results in a set of dynamical equations governing the behavior of the system that are invariant under rotation in charge space. Systems in which all particles have the same magnetic to electric charge ratio are indistinguishable from conventional electrodynamic systems consisting of ordinary electric charges and electromagnetic fields. However, systems consisting of particles with different charge ratios behave differently. It is shown that for a hydrogen-like atom consisting of two dual-charged particles with different charge ratios, some of the degeneracy within the hydrogen atom energy states is removed.

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LIST OF SYMBOLS

A_μ	four-dimensional vector potential
$\tilde{A}_\mu$	four-dimensional pseudovector potential
$\underline{A}$	vector potential
$\underline{a}$	acceleration
$\underline{B}$	magnetic field component
$\underline{B}^*$	generalized magnetic field component
$\hat{\underline{B}}$	charge independent magnetic field vector
c	speed of light
e	electric charge
$\tilde{e}$	magnetic charge
$\bar{e}^*$	dual-charged phasor
e*	magnitude of the dual-charged phasor
$\underline{E}$	electric field component
$\underline{E}^*$	generalized electric field component
$\hat{\underline{E}}$	charge independent electric field vector
E	energy
$\|E\rangle$	energy state
$F_{\mu\nu}$	electromagnetic field tensor
$\tilde{F}_{\mu\nu}$	dual electromagnetic field tensor
$\underline{G}$	Poynting vector
H	Hamiltonian
H_o	unperturbed Hamiltonian
H'	perturbation Hamiltonian
h	Planck's constant
I	electric charge current

I^*	dual-charged current
j_μ	four-dimensional electric current density
$\tilde{j}_\mu$	four-dimensional magnetic current density
$\underline{J}$	electric current density
$\underline{\tilde{J}}$	magnetic current density
$\underline{J}^*$	dual-charge current density
K	energy constant
m	mass
$\underline{n}$	unit vector
$\|nlm\rangle$	eigenstate of hydrogen atom
$\underline{p}$	momentum
P	power
$\bar{P}$	average power
$\underset{\sim}{R}$	two-dimensional charge-space rotation matrix
s	space interval
t	time
U_{nlm}	hydrogen atom wave function
$\underline{v}$	velocity
X_μ	four-dimensional space-time point
α	phase angle of dual-charge phasor
δ	difference in phase angles
$\delta^3(\underline{r}-\underline{r}_1)$	three-dimensional Dirac delta function
ε_o	permittivity of free space
$\varepsilon_{\mu\nu\sigma\rho}$	antisymmetric fourth-ranked unit tensor
ζ	constant
λ	linear electric-charge density
λ^*	linear dual-charge density

μ	reduced mass
μ_0	permeability of free space
ν	frequency
ρ	electric-charge density
$\tilde{\rho}$	magnetic-charge density
ρ^*	dual-charge density
σ	Thompson cross section
Φ	scalar potential
ω	angular frequency

I. INTRODUCTION

Although the study of electromagnetic phenomena is the best understood of all physical science, there are two very fundamental mysteries associated with it. The first is that the electromagnetic equations of Maxwell show an intrinsic symmetry between electric and magnetic quantities, but yet, there is no known magnetic counterpart to electric charge. The second is that the unit of electric charge is universal and is observed with fantastic precision to be identical on all charged particles despite variations in their other characteristics. These two mysteries were in essence answered by P. A. M. Dirac in 1948 in his paper on The Theory of Magnetic Monopoles [Ref. 1] in which, by using symmetry arguments, he hypothesized the existence of magnetic monopoles and thus mathematically symmetrized Maxwell's equations. These magnetic monopoles were elementary particles with magnetic charge analogous to the electric charge normally associated with conventionally charged particles. By assuming the coexistence of magnetic monopoles and electric charges interacting through the medium of the electromagnetic field, he was able to provide a complete dynamical theory. Then, through the laws of quantum mechanics, he also showed that this led to the requirement that the electric charge be quantized, i.e., that all charges must be integral multiples of a unit charge e connected with the magnetic monopole charge $\tilde{e}$ by the formula

$$e\tilde{e} = \frac{1}{2} 4\pi\varepsilon_0 \hbar c$$

As a result of this equation and the fine structure constant,

$$\frac{e^2}{4\pi\varepsilon_0 \hbar c} \cong \frac{1}{137}$$

then $\tilde{e}$ has the large value of 68.5e, which may account for the monopole's elusiveness in that it is tightly bound to its negative counterpart.

In 1969, Julian Schwinger, speculating on the structure of nuclear physics, proposed a new type of particle with both electric and magnetic charge. In his paper on A Magnetic Model of Matter [Ref. 2] he put forth a speculative hypothesis that electric and magnetic charge can reside on a single particle without violating any of the results given by Dirac. Therefore, it is the intention of this paper to further develop the set of classical equations used to describe systems of interacting dual-charged particles and electromagnetic fields and then to develop a quantum mechanical description of a hydrogen-like dual-charged atom.

II. THE SET OF CLASSICAL EQUATIONS GOVERNING THE BEHAVIOR OF A SYSTEM OF INTERACTING DUAL-CHARGED PARTICLES AND ELECTROMAGNETIC FIELDS

A. CONVENTIONAL ELECTRODYNAMIC THEORY

From conventional electrodynamic theory, the classical set of equations governing the behavior of a system of interacting electric charges and electromagnetic fields can be written in four-vector notation as

$$\partial^{\nu} F_{\mu\nu} = -\mu_0 c j_{\mu} \tag{II-1}$$

$$\partial_{\sigma} F_{\mu\nu} + \partial_{\mu} F_{\nu\sigma} + \partial_{\nu} F_{\sigma\mu} = 0 \tag{II-2}$$

$$mc \frac{d^2 x_{\mu}}{ds^2} = e F_{\mu\nu} \frac{dx^{\nu}}{ds} \tag{II-3}$$

where $F_{\mu\nu}$ is the antisymmetric electromagnetic field tensor defined in terms of the four-dimensional vector potential

$$A^{\mu} = \left(\frac{\Phi}{c} - \underline{A} \right)$$

as

$$F_{\mu\nu} \equiv \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \tag{II-4}$$

with Φ and $\underline{A}$ being the three-dimensional scalar and vector potentials used to describe the electromagnetic field

$$\underline{E} = -\nabla\Phi - \partial\underline{A}/\partial t \tag{II-5}$$

$$\underline{B} = \nabla \times \underline{A} \tag{II-6}$$

In terms of $\underline{E}$ and $\underline{B}$, $F_{\mu\nu}$ can be written as

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

with Eqs. II-1 through II-5 written equivalently as

$$\nabla \cdot \underline{E} = \mu_0 c^2 \rho \tag{II-7}$$

$$\nabla \times \underline{B} - \frac{1}{c} \partial \underline{E}/\partial t = \mu_0 \underline{J} \tag{II-8}$$

$$\nabla \cdot \underline{B} = 0 \tag{II-9}$$

$$\nabla \times \underline{E} + \partial \underline{B}/\partial t = 0 \tag{II-10}$$

$$dp/dt = e\left(\underline{E} + \underline{v} \times \underline{B}\right) \tag{II-11}$$

Equations II-7 through II-10 are known as Maxwell's equations and Eq. II-11 is known as the Lorentz Force Law for an electric charge in an external electromagnetic field.

For expediency in further analysis, it will be convenient to express Eqs. II-7 through II-11 in matrix notation, namely

$$\begin{pmatrix} -\frac{1}{c}\partial/\partial t & \nabla \times \\ -\nabla \times & -\frac{1}{c}\partial/\partial t \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c \begin{pmatrix} \underline{J} \\ 0 \end{pmatrix} \tag{II-12}$$

$$\nabla \cdot \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c^2 \begin{pmatrix} \rho \\ 0 \end{pmatrix} \qquad \text{(II-13)}$$

$$d\psi/dt = \begin{pmatrix} 1 & \frac{1}{c}\hat{\nabla}\times \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix}$$

Written in matrix notation, Maxwell's equations show a definite lack of symmetry; i.e., the zeros on the r.h.s. of Eqs. II-12 and II-13. This asymmetry is even more apparent if one rewrites Eq. II-2 in terms of the dual electromagnetic field tensor $\tilde{F}_{\mu\nu}$ defined by

$$\tilde{F}_{\mu\nu} \equiv -\tfrac{1}{2}\,\varepsilon_{\mu\nu}{}^{\sigma\rho}\,F_{\sigma\rho} \qquad \text{(II-14)}$$

where $\varepsilon_{\mu\nu\sigma\rho}$ is the completely antisymmetric unit fourth-ranked tensor. One has

$$\begin{aligned}
\partial^\nu \tilde{F}_{\mu\nu} &= -\tfrac{1}{2}\,\varepsilon_{\mu\nu}{}^{\sigma\rho}\,\partial^\nu F_{\sigma\rho} \\
&= -\tfrac{1}{2}\,\varepsilon_{\mu\nu}{}^{\sigma\rho}\left(\partial^\nu \partial_\sigma A_\rho - \partial^\nu \partial_\rho A_\sigma\right) \\
&= -\tfrac{1}{2}\left(\varepsilon_{\mu\nu}{}^{\rho\sigma} - \varepsilon_{\mu\nu}{}^{\sigma\rho}\right)\partial^\nu \partial_\rho A_\sigma \\
&= -\varepsilon_{\mu\nu}{}^{\rho\sigma}\,\partial^\nu \partial_\rho A_\sigma \\
&= 0
\end{aligned}$$

since $\partial_\nu \partial_\rho$ is symmetric and $\varepsilon_{\mu\nu\rho\sigma}$ is antisymmetric under the interchange $\nu \leftrightarrow \rho$. Hence Eq. II-2 can be written as

$$\partial^\nu \tilde{F}_{\mu\nu} = 0 \tag{II-15}$$

with Maxwell's equations, in four-vector matrix notation, given as

$$\partial^\nu \begin{pmatrix} F_{\mu\nu} \\ \tilde{F}_{\mu\nu} \end{pmatrix} = -\mu_0 c \begin{pmatrix} j_\mu \\ 0 \end{pmatrix} \tag{II-16}$$

in terms of $\underline{E}$ and $\underline{B}$,

$$\tilde{F}_{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & E_z/c & -E_y/c \\ -B_y & -E_z/c & 0 & E_x/c \\ -B_z & E_y/c & -E_x/c & 0 \end{pmatrix} \tag{II-17}$$

The lack of mathematical symmetry in Eq. II-16 is readily apparent in that while the tensor $F_{\mu\nu}$ has the source term j_μ, the dual tensor $\tilde{F}_{\mu\nu}$ does not. This lack of symmetry can be easily rectified simply by giving $\tilde{F}_{\mu\nu}$ an equivalent source term, namely $\tilde{j}_\mu$, replacing Eq. II-15 with

$$\partial^\nu \tilde{F}_{\mu\nu} = -\mu_0 c \tilde{j}_\mu \tag{II-18}$$

However, by giving $\tilde{F}_{\mu\nu}$ a source, then $F_{\mu\nu}$ can no longer be defined in terms of the four-dimensional vector potential A^μ as given by Eq. II-4, since this would ultimately lead back to Eq. II-15.

Before dwelling on the physical significance of mathematically symmetrizing Maxwell's equations, it would perhaps be useful to re-examine some of the fundamental beliefs currently held in conventional electrodynamic theory; namely, those with respect to the relationships between point charges and electromagnetic fields. Classically, in studying the interaction of charged particles, it is convenient to develop the concept of a field of force and instead of saying that one particle acts on another, one says that each charged particle creates a field of force about itself, this force then acting on every other particle in the field. Classically, the field is merely a mode of description of the physical phenomenon; namely, the interaction of particles. However, from the theory of relativity, interactions propagate at a finite velocity and as a result, the field itself acquires a physical reality. One cannot speak of direct interactions between particles separated in space; one must speak of particles creating physical fields and the subsequent interaction of these fields with other particles.

Since it is Maxwell's dynamic field equation, Eq. II-1, which establishes the relationship between the source charge and the resultant electromagnetic field, one can easily determine the electromagnetic field of an accelerating point electric charge of charge magnitude e to be

$$\underline{E} = \frac{e\mu_0 g^3}{4\pi s}\left\{\frac{c^2}{s}\left(1-\frac{v^2}{c^2}\right)\left(\underline{n}-\underline{v}/c\right) + \underline{n}\times\left[\left(\underline{n}-\underline{v}/c\right)\times\underline{a}\right]\right\} \qquad \text{(II-19)}$$

$$c\underline{B} = \underline{n}\times\underline{E} \qquad \text{(II-20)}$$

where $\underline{v}$ is the particle's velocity, $\underline{a}$ its acceleration, s the space interval from the particle to the field point, $\underline{n}$ the unit vector along s and

$$g = \frac{1}{1-\frac{1}{c}\underline{n}\cdot\underline{v}}$$

with all quantities determined at the retarded time. Inherent in such an analysis is that $F_{\mu\nu}$ is defined in terms of A^{μ} by Eq. II-4; then utilizing the retarded Green's function to determine the Lienard-Wiechert potentials, one uses Eqs. II-5 and II-6 to obtain the resultant electromagnetic field. The significant aspect of the fields thus derived is that $\underline{E}$, the electric field, can exist in the absence of the particle's motion, whereas the magnetic field $\underline{B}$ cannot. The magnetic field is due to electric charges in motion. However, due to the transformation of the electromagnetic fields under Lorentz transformations, it can easily be shown that $\underline{E}$ and $\underline{B}$ have no independent existence. The fields are completely interrelated; for what is a purely electric or magnetic field in one coordinate system will appear as a mixture of electric and magnetic fields in another coordinate frame. Certain restrictions apply, the significant one here being that a purely electrostatic field in one coordinate system cannot be transformed into a purely magnetostatic field in another. Therefore,

one should properly speak of the electromagnetic field tensor, $F_{\mu\nu}$, rather than $\underline{E}$ or $\underline{B}$ separately. However, the significance of symmetrizing Maxwell's equations significantly alters these conceptions as the ensuing analysis will show.

B. DIRAC'S THEORY OF MAGNETIC MONOPOLES

The first attempt at symmetrizing Maxwell's equations was given by P. A. M. Dirac [Ref. 1]. Mathematically, symmetrizing Maxwell's equation is rather trivial. One assumes an equivalent source term $\tilde{j}_\mu$ for the dual electromagnetic field tensor $\tilde{F}_{\mu\nu}$ and rewrites Maxwell's equations as

$$\partial^\nu \begin{pmatrix} F_{\mu\nu} \\ \tilde{F}_{\mu\nu} \end{pmatrix} = -\mu_0 c \begin{pmatrix} j_\mu \\ \tilde{j}_\nu \end{pmatrix} \tag{II-21}$$

In analogy with the four-dimensional electric current density

$$j_\mu \equiv \left(\rho, \frac{1}{c}\underline{J}\right)$$

one can simply denote the components of the four-dimensional magnetic current density by

$$\tilde{j}_\mu \equiv \left(\tilde{\rho}, \frac{1}{c}\underline{\tilde{J}}\right)$$

where the choice of the word "magnetic" is the result of hindsight rather than mathematical logic. One could define $\tilde{\rho}$ and $\underline{\tilde{J}}$ as the magnetic charge and current densities respectively. Since $\tilde{F}_{\mu\nu}$ is, by Eq. II-14, a pseudotensor, it follows that $\tilde{j}_\mu$ must be a pseudovector in order that Eq. II-18 be Lorentz covariant. In addition, since $\tilde{F}_{\mu\nu}$ is antisymmetric, taking the divergence of Eq. II-18 gives

$$\partial^\mu j_\mu = 0$$

so that in this model described by Eq. II-21, both electric and magnetic charges are independently conserved. From Eq. II-17, one can equivalently express Eq. II-18 in terms of E and B, giving

$$\nabla \cdot \underline{B} = \mu_0 c \tilde{\rho}$$

$$\nabla \times \underline{E} + \partial \underline{B}/\partial t = -\mu_0 c \tilde{\underline{J}}$$

Therefore, Maxwell's equations, in terms of $\underline{E}$ and $\underline{B}$, become

$$\begin{pmatrix} -\frac{1}{c}\partial/\partial t & \nabla \times \\ -\nabla \times & -\partial/\partial t \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c \begin{pmatrix} \underline{J} \\ \tilde{\underline{J}} \end{pmatrix} \qquad \text{(II-22)}$$

$$\nabla \cdot \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c^2 \begin{pmatrix} \rho \\ \tilde{\rho} \end{pmatrix} \qquad \text{(II-23)}$$

The physical significance of symmetrizing Maxwell's equations is that one is compelled to hypothesize the existence of magnetically charged particles; i.e., magnetic monopoles, that can coexist with electric charges. Maxwell's equations, in describing the relationship between these two

independent types of charges and the resultant electromagnetic field, redefine the exact nature of the magnetic field in that both the resultant electric and magnetic fields of the system are equivalent in all aspects; namely, they can both be produced by static charges, as well as charges in motion. Although the coexistence of electric charges and magnetic monopoles is physically plausible, it ultimately leads to significant mathematical difficulties in that one cannot use indiscriminately the four-dimensional vector potential A^{μ} to describe the resultant electromagnetic field of this system. One is forced to developed unphysical variables; i.e., Dirac strings [Ref. 1] in order to perform any further mathematical analysis. This problem will be encountered in the quantum mechanical analysis in a later section.

In order to determine the exact nature of the magnetic monopole hypothesized by symmetrizing Maxwell's equations, consider a system consisting solely of magnetic monopoles and the resultant electromagnetic field due to these monopoles. Maxwell's equations governing the behavior of this system are given by

$$\partial^{\nu}\begin{pmatrix} F_{\mu\nu} \\ \tilde{F}_{\mu\nu} \end{pmatrix} = -\mu_0 c \begin{pmatrix} 0 \\ \tilde{j}_{\mu} \end{pmatrix} \tag{II-24}$$

which in terms of $\underline{E}$ and $\underline{B}$ are

$$\begin{pmatrix} -\frac{1}{c}\partial/\partial t & \nabla\times \\ -\nabla\times & -\frac{1}{c}\frac{\partial}{\partial t} \end{pmatrix}\begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c \begin{pmatrix} 0 \\ \tilde{\underline{j}} \end{pmatrix} \tag{II-25}$$

$$\nabla\cdot\begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c^2 \begin{pmatrix} 0 \\ \tilde{\rho} \end{pmatrix} \tag{II-26}$$

One could in theory define a four-dimensional pseudovector potential

$$\tilde{A}^{\mu} \equiv \left(\frac{\tilde{\Phi}}{c} , \underline{\tilde{A}} \right)$$

and redefine $\tilde{F}_{\mu\nu}$ in terms of $\tilde{A}^{\mu}$ instead of $F_{\mu\nu}$ as

$$\tilde{F}_{\mu\nu} \equiv \partial_{\mu} \tilde{A}_{\nu} - \partial_{\nu} \tilde{A}_{\mu}$$

and utilizing the analysis of conventional electrodynamic theory, ultimately derive Eq. II-24 and an equivalent differential equation of motion for a magnetic monopole in the external electromagnetic field of this system as

$$mc \frac{d^2 x_{\mu}}{ds^2} = \tilde{e} \tilde{F}_{\mu\nu} \frac{dx^{\nu}}{ds} \qquad \text{(II-27)}$$

where $\tilde{e}$, a pseudoscalar, is the magnetic charge. From Eq. II-17, one obtains an equivalent Lorentz Force Law for a magnetic monopole as

$$dp/dt = \tilde{e} \left(c\underline{B} - \frac{1}{c} \underline{v} \times \underline{E} \right) \qquad \text{(II-28)}$$

Equations II-24 and II-27 govern the behavior of a system of magnetic monopoles in the same manner as Eqs. II-3 and II-16 govern the behavior of a system of electric charges. Since the governing equations are identical in form, the reactions within each system are indistinguishable from one another. This implies that, given equivalent conditions, two interacting magnetic monopoles should behave in a manner identical to two interacting electric charges. Since the mode of interaction is via the electromagnetic field, it

readily follows that the magnetic field of magnetic monopoles should be equivalent to the electric field of an electric charge, whereas the electric field of a magnetic monopole would be equivalent to the magnetic field of an electric charge. This will be borne out in a later section.

However, in the case of a system consisting of both electrically charged particles and magnetic monopoles, the situation is quite different. The interaction between an electric charge and a magnetic monopole differs from that between two electric charges. Rather than dwell on how this interaction differs at this time, it will be more beneficial to further generalize the situation with the introduction of dual-charged particles in the next section.

C. SCHWINGER'S THEORY OF DUAL-CHARGED PARTICLES

Julian Schwinger modified Dirac's theory of magnetic monopoles by introducing particles having both electric and magnetic charge [Ref 2]. If one hypothesizes the existence of such particles, then from Eqs. II-11 and II-28, the force on such a particle in an external electromagnetic field is given by

$$d\underline{p}/dt = e\left(\underline{E} + \underline{v}\times\underline{B}\right) + \tilde{e}\left(c\underline{B} - \frac{1}{c}\underline{v}\times\underline{E}\right)$$

which in matrix notation can be written as

$$\frac{dp}{dt} = \begin{pmatrix} e & \tilde{e} \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{c}\nabla\times \\ -\frac{1}{c}\nabla\times & 1 \end{pmatrix} \begin{pmatrix} E \\ cB \end{pmatrix} \tag{II-29}$$

One method of simplifying Eq. II-29 is to redefine the electric and magnetic charge associated with this particle in terms of a composite charge phasor $\bar{e}^*$, shown diagrammatically in Fig. II-1.

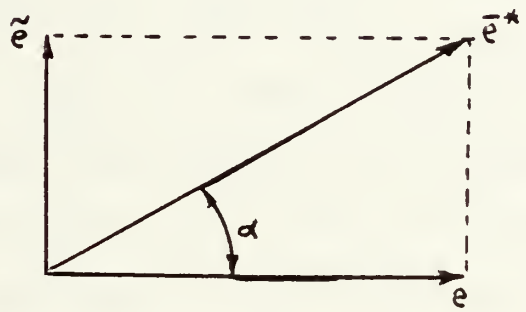

Figure II-1

By defining the phasor such that

$$\bar{e}^* = e^* e^{i\alpha} \tag{II-30}$$

where

$$e^* = (e^2 + \tilde{e}^2)^{1/2} \tag{II-31}$$

$$\alpha = \tan^{-1} \tilde{e}/e \tag{II-32}$$

then it readily follows that

$$e = e^* \cos\alpha \tag{II-33}$$

$$\tilde{e} = e^* \sin\alpha \tag{II-34}$$

Substituting for (e e), Eq. II-29 becomes

$$\frac{dp}{dt} = e^* \begin{pmatrix} \cos\alpha & \sin\alpha \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{c}\nabla x \\ -\frac{1}{c}\nabla x & 1 \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix}$$

$$= e^* \begin{pmatrix} 1 & \frac{1}{c}\nabla x \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} \tag{II-35}$$

By use of the composite charge phasor, one can establish a two-dimensional charge space at all points in four-dimensional space-time to describe the charge distribution for a given system. By adopting this two-dimensional charge space concept, one can then define a more generalized charged particle; i.e., a dual-charged particle which can have either electric, magnetic or a combination of both electric and magnetic charge. The charge of such a particle is the composite charge phasor $\vec{e}$ given by Eq. II-30, which has a charge magnitude e^* given by Eq. II-31 and phase α, denoting the ratio of magnetic charge to electric charge, given by Eq. II-32. Thus, by adopting this notation, one should be able to also rework Maxwell's equation for a system of coexisting electric charges and magnetic monopoles into a more general set of equations describing a system of interacting dual-charged particles.

To see how this comes about, consider a system consisting of n dual-charged point particles having electric charge e_i, magnetic charge $\tilde{e}_i$, positions $\underline{r}_i$ and velocities $\underline{v}_i$. From Eqs. II-25 and II-26, it is readily apparent that the field equations for this system are given by

$$\begin{pmatrix} -\frac{1}{c}\partial/\partial t & \nabla\times \\ -\nabla\times & -\frac{1}{c}\partial/\partial t \end{pmatrix}\begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c \sum_{i=1}^{n}\begin{pmatrix} e_i \\ \tilde{e}_i \end{pmatrix}\underline{v}_i\,\delta^3(\underline{r}-\underline{r}_i) \qquad \text{(II-36)}$$

$$\nabla\cdot\begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c^2 \sum\begin{pmatrix} e_i \\ \tilde{e}_i \end{pmatrix}\delta^3(\underline{r}-\underline{r}_i) \qquad \text{(II-37)}$$

Substituting for e and $\tilde{e}$ from Eqs. II-33 and II-34, Eqs. II-36 and II-37 become

$$\begin{pmatrix} -\frac{1}{c}\partial/\partial t & \nabla\times \\ -\nabla\times & -\frac{1}{c}\partial/\partial t \end{pmatrix}\begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c \sum_{i=1}^{n} e_i^* \begin{pmatrix} \cos\alpha_i \\ \sin\alpha_i \end{pmatrix}\underline{v}_i\,\delta^3(\underline{r}-\underline{r}_i) \qquad \text{(II-38)}$$

$$\nabla\cdot\begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c^2 \sum_{i=1}^{n} e_i^* \begin{pmatrix} \cos\alpha_i \\ \sin\alpha_i \end{pmatrix}\delta^3(\underline{r}-\underline{r}_i) \qquad \text{(II-39)}$$

Now suppose that all of the particles in this system have the same charge ratio $\tilde{e}_i/e_i$; that is

$$\alpha_i = \alpha$$

for all i, and noting that the two-dimensional rotation matrix

$$\underset{\sim}{R} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \tag{II-40}$$

commutes with the differential operators on the l.h.s. of Eqs. II-38 and II-39, one finds that

$$\begin{pmatrix} -\frac{1}{c}\partial/\partial t & \nabla\times \\ -\nabla\times & -\frac{1}{c}\partial/\partial t \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sum_{i=1}^{n} e_i^* \underline{v}_i \delta^3(\underline{r}-\underline{r}_i)$$

$$\nabla\cdot \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sum_{i=1}^{n} e_i^* \delta^3(\underline{r}-\underline{r}_i)$$

Defining the current charge densities of the system by

$$\underline{J}^* = \sum_{i=1}^{n} e_i^* \underline{v}_i \delta^3(\underline{r}-\underline{r}_i) \tag{II-41}$$

$$\rho^* = \sum_{i=1}^{n} e_i^* \delta^3(\underline{r}-\underline{r}_i) \tag{II-42}$$

the field equations become

$$\begin{pmatrix} -\frac{1}{c}\partial/\partial t & \nabla\times \\ -\nabla\times & -\frac{1}{c}\partial/\partial t \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c \begin{pmatrix} \underline{J}^* \\ 0 \end{pmatrix}$$

$$\nabla\cdot \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c^2 \begin{pmatrix} \rho^* \\ 0 \end{pmatrix}$$

Finally, defining new fields

$$\begin{pmatrix} \underline{E}' \\ c\underline{B}' \end{pmatrix} \equiv \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} \tag{II-43}$$

the set of equations describing the behavior of a system of dual-charged particles, all having the same α, is given by

$$\begin{pmatrix} -\frac{1}{c}\frac{\partial}{\partial t} & \nabla\times \\ -\nabla\times & -\frac{1}{c}\frac{\partial}{\partial t} \end{pmatrix} \begin{pmatrix} \underline{E}' \\ c\underline{B}' \end{pmatrix} = \mu_0 c \begin{pmatrix} \underline{J}' \\ 0 \end{pmatrix} \tag{II-44}$$

$$\nabla\cdot \begin{pmatrix} \underline{E}' \\ c\underline{B}' \end{pmatrix} = \mu_0 c^2 \begin{pmatrix} \rho' \\ 0 \end{pmatrix} \tag{II-45}$$

$$\frac{d\rho}{dt} = e^* \left(1 \quad \frac{1}{c}\nabla\times \right) \begin{pmatrix} \underline{E}' \\ c\underline{B}' \end{pmatrix} \tag{II-46}$$

the last equation following from Eq. II-35. Equations II-44 through II-46 are the generalized set of equations desired. One should note with special emphasis that all reference to the phase angle α has disappeared from Eqs. II-44 through II-46, and that these equations describe conventional electrodynamics. Hence, systems of dual-charged conventional particles in which all particles have the same α are indistinguishable from conventional electrodynamics.

The significant aspect of this development is that Maxwell's equations and the Lorentz Force Law are invariant under rotation in two-dimensional charge space. The charges (e $\tilde{e}$) and the fields ($\underline{E}$,$c\underline{B}$) transform as vectors in this charge space and the matrix $\underset{\sim}{R}$ as defined in Eq. II-40 is the rotation matrix that operates on these vectors. A rotation of the system through the angle α leaves the governing

equations invariant and hence the system is indistinguishable from the original system.

This invariance makes it quite easy to determine the various characteristics of dual-charged particles. One simply needs to know the characteristics attributed to electric charges in conventional electrodynamic systems and perform suitable charge-space rotations on the result. This shall be the subject of the next section.

III. GENERAL CHARACTERISTICS OF DUAL-CHARGED PARTICLES

A. THE ELECTROMAGNETIC FIELD OF AN ACCELERATING DUAL-CHARGED PARTICLE

The electromagnetic field of a point dual-charged particle of charge ratio α can be found by rotating (in charge-space) Maxwell's equations for a point electric charge through an angle α. The procedure is the reverse of that which led from Eqs. II-36 and II-37 to Eqs. II-44 and II-45. For a single point electric charge e* having position $\underline{r}_1$ and velocity $\underline{v}_1$, from Eqs. II-41 and II-42, the charge and current densities are given by

$$\rho^* = e^* \delta^3(\underline{r}-\underline{r}_1) \tag{III-1}$$

$$\underline{J}^* = e^* \underline{v}_1 \delta^3(\underline{r}-\underline{r}_1) \tag{III-2}$$

Putting Eqs. III-1 and III-2 into Eqs. II-44 and II-45 and working backwards to Eqs. II-36 and II-37, one obtains

$$\begin{pmatrix} \frac{1}{c}\partial/\partial t & \nabla\times \\ -\nabla\times & -\frac{1}{c}\partial/\partial t \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c \begin{pmatrix} e \\ \tilde{e} \end{pmatrix} \underline{v}_1 \delta^3(\underline{r}-\underline{r}_1)$$

$$\nabla \cdot \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \mu_0 c^2 \begin{pmatrix} e \\ \tilde{e} \end{pmatrix} \delta^3(\underline{r}-\underline{r}_1)$$

with e and $\tilde{e}$ related to e* via Eqs. II-33 and 34 and the fields $(\underline{E},\underline{B})$, created by the dual-charged particle, related to the fields $(\underline{E}^*, c\underline{B}^*)$, created by an electric charge e*, via Eq. II-43. Equation II-43 can be inverted to yield

$$\begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \underline{E}^* \\ c\underline{B}^* \end{pmatrix} \qquad \text{(III-3)}$$

Since $\underline{E}^*$ and $\underline{B}^*$ are known, and, in fact, are precisely the fields given by Eqs. II-19 and II-20 (with e replaced by e*), Eq. III-3 determines the electromagnetic field of a dual-charged particle of charge magnitude e* and charge ratio α. For the sake of convenience in subsequent analysis, define the following charge independent electromagnetic field vectors of an accelerating dual-charged particle:

$$\hat{\underline{E}} \equiv \frac{1}{e^*}\underline{E}$$

$$\hat{\underline{B}} \equiv \frac{1}{e^*}\underline{B}$$

Then, utilizing this notation, the electromagnetic field of an accelerating dual-charged particle is given by

$$\begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = e^* \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \hat{\underline{E}} \\ c\hat{\underline{B}} \end{pmatrix} \qquad \text{(III-4)}$$

where

$$\hat{\underline{E}} = \frac{\mu_0 q^3}{4\pi s}\left\{\frac{c^2}{s}\left(1-\frac{v^2}{c^2}\right)\left(\underline{n}-\underline{v}/c\right) + n\times\left[\left(\underline{n}-\underline{v}/c\right)\times\underline{a}\right]\right\}$$

$$c\hat{\underline{B}} = \underline{n}\times\hat{\underline{E}}$$

B. RADIATION CHARACTERISTICS OF MOVING DUAL-CHARGED PARTICLES

The Poynting vector of an electromagnetic field is

$$\underline{G}(\underline{r},t) = \frac{1}{\mu_0 c}\left(\underline{E} \times c\underline{B}\right)$$

Using Eq. III-4, one finds that the Poynting vector for the fields radiated from an accelerating dual-charged particle is

$$\underline{G}(\underline{r},t) = \frac{e^{*2}}{\mu_0 c}\left[\left(\cos\alpha\,\hat{\underline{E}} - \sin\alpha\, c\hat{\underline{B}}\right) \times \left(\sin\alpha\,\hat{\underline{E}} + \cos\alpha\, c\hat{\underline{B}}\right)\right]$$

$$= \frac{e^{*2}}{\mu_0 c}\left[\cos^2\alpha\left(\hat{\underline{E}} \times c\hat{\underline{B}}\right) - \sin^2\alpha\left(c\hat{\underline{B}} \times \hat{\underline{E}}\right)\right]$$

$$= \frac{e^{*2}}{\mu_0 c}\left(\hat{\underline{E}} \times c\hat{\underline{B}}\right) \qquad \text{(III-5)}$$

where it should be noted that $\hat{\underline{E}} \perp c\hat{\underline{B}}$. Since a subsequent analysis of Eq. III-5 involves the vectors $\hat{\underline{E}}$ and $c\hat{\underline{B}}$, with charge being treated as a constant, it can easily be verified by conventional electrodynamic theory that the power radiated by the dual-charged particle in the non-relativistic limit is

$$P(t) = \frac{2}{3}\,\frac{e^{*2}}{4\pi\varepsilon_0}\,\frac{a^2(t)}{c^3} \qquad \text{(III-6)}$$

Equation III-6 results in the same radiation reaction for dual-charged particles as for ordinary electrically charged particles.

C. REACTIONS OF DUAL-CHARGED PARTICLES IN EXTERNAL ELECTROMAGNETIC FIELDS

The reaction of a dual-charged particle in an external electromagnetic field is governed by the Lorentz Force Law

$$d\underline{p}/dt = e^* \begin{pmatrix} 1 & \frac{1}{c}\underline{v}\times \end{pmatrix} \begin{pmatrix} \underline{E}^* \\ c\underline{B}^* \end{pmatrix} \qquad \text{(II-46)}$$

where

$$\begin{pmatrix} \underline{E}^* \\ c\underline{B}^* \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} \qquad \text{(II-43)}$$

with $\underline{E}$ and $\underline{B}$ being the external electromagnetic field components themselves. In an external electromagnetic field $(\underline{E}, c\underline{B})$, the dual-charged particle "sees" the field as $(\underline{E}^*, c\underline{B}^*)$, not as $(\underline{E}, c\underline{B})$, where from Eq. II-43

$$\underline{E}^* = \cos\alpha\, \underline{E} + \sin\alpha\, c\underline{B}$$

$$c\underline{B}^* = -\sin\alpha\, \underline{E} + \cos\alpha\, c\underline{B}$$

Since $\underline{E}$ and $c\underline{B}$ are vector fields, with $\cos\alpha$ and $\sin\alpha$ being fixed numbers, then $\underline{E}^*$ and $c\underline{B}^*$ are vector fields which produce the same effect on e^* that $\underline{E}$ and $c\underline{B}$ produce on an electric charge e. In other words, e^* responds in an identical manner to $\underline{E}^*$ and $c\underline{B}^*$ that e responds to $\underline{E}$ and $c\underline{B}$; particularly, a dual-charged particle is accelerated by $\underline{E}^*$ and has its trajectory bent by $c\underline{B}^*$.

Consider an electric charge in an external electromagnetic field, which is plane and linearly polarized and traveling in the z-direction, given by

$$\underline{E} = E_0 \cos\omega(t - \tfrac{1}{c} z)\,\hat{i}$$

$$c\underline{B} = E_0 \cos\omega(t - \tfrac{1}{c} z)\,\hat{j}$$

where $\hat{i}$ and $\hat{j}$ are unit vectors in the x and y-directions. The force on the electric charge is given by Eq. II-11, which in the non-relativistic limit reduces to

$$m \frac{d^2x}{ds^2} = e E_0 \cos\omega t \qquad \text{(III-7)}$$

where, since there are no forces in the z-direction, z remains constant (if $\dot{z}=0$) and is set equal to zero. The solution to Eq. III-7 is

$$x(t) = x_0 + v_x(0)\, t - \frac{e E_0}{m\omega^2} \cos\omega t$$

Thus, the particle oscillates harmonically in the x-direction. The average power radiated is

$$\bar{P} = \frac{1}{3} \frac{e^4}{4\pi\varepsilon_0} \frac{E_0^2}{m^2 c^3} \qquad \text{(III-8)}$$

The total scattering cross-section, σ , is given by

$$\sigma = \frac{\bar{P}}{|G_{inc}|}$$

where G_{inc} is the incident electromagnetic flux, given by

$$G_{inc} = \frac{E_0^2}{2\mu_0 c}$$

Therefore,

$$\sigma = \frac{8\pi}{3}\left[\frac{e^2/4\pi\varepsilon_0}{mc^2}\right]^2 \qquad \text{(III-9)}$$

which is the Thompson cross-section for an electric charge.

If one now considers a dual-charged particle instead of the electric charge, an identical analysis could be given if one takes

$$\underline{E}^* = E_0 \cos\omega\left(t - \tfrac{1}{c}z\right)\hat{i} \qquad \text{(III-10)}$$

$$c\underline{B}^* = E_0 \cos\omega\left(t - \tfrac{1}{c}z\right)\hat{j} \qquad \text{(III-11)}$$

The dual-charged particle oscillates harmonically in the x-direction with the average power radiated and the Thompson cross-section given by

$$\bar{P} = \frac{1}{3}\frac{e^{*4}}{4\pi\varepsilon_0}\frac{E_0^2}{m^4c^3} \qquad \text{(III-12)}$$

$$\sigma = \frac{8\pi}{3}\left[\frac{e^{*2}/4\pi\varepsilon_0}{mc^2}\right]^2 \qquad \text{(III-13)}$$

As anticipated, the only differences between Eqs. III-8, 9 and III-12, 13 are e* vice e. However, $\underline{E}^*$ and $c\underline{B}^*$ are not the actual fields of the system. The actual fields of the system are related to $\underline{E}^*$ and $c\underline{B}^*$ by Eq. III-3, which upon substituting Eqs. III-10 and III-11 gives

$$\begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = E_0 \cos\omega\left(t - \tfrac{1}{c}z\right)\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \hat{i} \\ \hat{j} \end{pmatrix}$$

If one defines the unit vectors $\hat{e}_1$ and $\hat{e}_2$ to be directed along $\underline{E}$ and $c\underline{B}$, then

$$\begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \hat{i} \\ \hat{j} \end{pmatrix}$$

from which it can be easily shown that $\hat{e}_1$ and $\hat{e}_2$ are orthonormal. Thus, to produce the desired $\underline{E}^*$ and $c\underline{B}^*$, the true electromagnetic field of the system must be oriented along $(\hat{e}_1, \hat{e}_2)$ and of equal magnitude to $(\underline{E}^*, c\underline{B}^*)$. This corresponds to simply rotating $(\underline{E}^*, c\underline{B}^*)$ through the angle $-\alpha$ about the z-axis. Thus, in this instance, a rotation of the system in charge space that leaves the system invariant involves a simple rotation of the electromagnetic field in ordinary space.

IV. SYSTEMS OF DUAL-CHARGED PARTICLES WITH UNIFORM CHARGE RATIOS

Rather than attempting to rewrite conventional electrodynamic theory in terms of the composite dual-charged particle e* and the generalized fields ($\underline{E}^*, c\underline{B}^*$), it is more convenient to consider just a few particular examples to further illustrate how such a task could be accomplished. The examples to be considered are relatively trivial; however, the intent is to amplify the presciption for proceeding from a conventional electrodynamic solution in a general dual-charged solution and to provide more insight into the physical significance of a rotated system. In general, the prescription will be to start with a solution in which

$$\begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = e \begin{pmatrix} \hat{\underline{E}} \\ c\hat{\underline{B}} \end{pmatrix} \tag{III-8}$$

and construct a solution where

$$\begin{pmatrix} \underline{E}^* \\ c\underline{B}^* \end{pmatrix} = e^* \begin{pmatrix} \hat{\underline{E}} \\ c\hat{\underline{B}} \end{pmatrix} \tag{III-9}$$

simply by replacing e by e* everywhere. The resultant electric and magnetic fields are then obtained by the equation

$$\begin{pmatrix} \underline{E} \\ c\underline{B} \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \underline{E}^* \\ c\underline{B}^* \end{pmatrix} \tag{III-3}$$

A. THE ELECTROMAGNETIC FIELD OF AN INFINITELY LONG, DUAL-CHARGED LINE

If one considers the electromagnetic field produced by a uniform electric charge distribution along an infinitely long, straight line, then it can be shown that the resultant electromagnetic field is an electrostatic field, given in polar coordinates as

$$\underline{E} = \frac{\lambda}{2\pi\varepsilon r}\hat{r}$$

where λ is the linear electric charge density. Therefore, according to the prescription, the resultant electromagnetic field is obtained by replacing λ by λ^* (the linear charge density for a dual-charge distribution) resulting in the static field

$$\underline{E}^* = \frac{\lambda^*}{2\pi\varepsilon_0 r}\hat{r}$$

where, from Eq. III-3, the actual electric and magnetic fields created by this dual-charge line are

$$\underline{E} = \cos\alpha \frac{\lambda^*}{2\pi\varepsilon r}\hat{r}$$

$$c\underline{B} = \sin\alpha \frac{\lambda^*}{2\pi\varepsilon r}\hat{r}$$

Therefore, a dual-charged particle placed at rest at $\underline{r}$ will experience a force directed along $\hat{r}$, the magnitude of this force depending upon the charge ratio of the dual-charged particle.

B. THE ELECTROMAGNETIC FIELD OF AN INFINITELY LONG, DUAL-CHARGED CURRENT

If one considers the magnetic field produced at an arbitrary point P by a straight, infinitely long, conductor carrying a current I consisting of moving electric charges, then in polar coordinates

$$\underline{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$

where $\hat{\phi}$ is the unit vector in the azimuthal direction in the plane perpendicular to the conductor with r being the radial distance from the conductor to the point P. By considering a dual-charged conduction current I* moving in a conductor made up of dual-charges (all particles in the system must have the same charge ratio α), but in which the net charge (both electric and magnetic) is zero, the resultant field is given by

$$\underline{B}^* = \frac{\mu_0 I^*}{2\pi r} \hat{\phi}$$

where the actual electric and magnetic fields are

$$\underline{E} = -\sin\alpha \frac{c\mu_0 I^*}{2\pi r} \hat{\phi}$$

$$\underline{B} = \cos\alpha \frac{\mu_0 I^*}{2\pi r} \hat{\phi}$$

V. SYSTEMS OF DUAL-CHARGED PARTICLES WITH NON-UNIFORM CHARGE RATIOS

The results obtained thus far differ from conventional electrodynamics only in how one defines the charges and associated fields in systems in which the particles all have the same charge ratio. As consistently demonstrated, the observable aspects of these systems are all independent of α. However, such is decidedly not the case in systems with non-uniform α distributions; that is, systems of particles having different charge ratios. Such systems are indeed unique and distinguishable from other systems and are uniquely described by Eqs. II-22, II-23 and II-29. In analyzing systems with non-uniform α distributions, it is imperative that one maintains a precise relationship between a given dual-charged particle and its resultant field. This will be borne out in the following analysis, which shall concern itself primarily with the interaction between two dual-charged particles with different charge ratios.

If one considers two dual-charged particles with different charge ratios α_1, and α_2, then starting with the Lorentz Force Law,

$$d\underline{P}_1/dt = e_1^{\wedge}\left(1 \quad \tfrac{1}{c}\underline{v}_1^{\times}\right)\begin{pmatrix}\cos\alpha_1 & \sin\alpha_1\\ -\sin\alpha_1 & \cos\alpha_1\end{pmatrix}\begin{pmatrix}\underline{E}_2\\ c\underline{B}_2\end{pmatrix} \qquad \text{(II-35)}$$

one has, upon substitution for the fields $(\underline{E}_2, c\underline{B}_2)$,

$$\begin{pmatrix} \underline{E}_2 \\ c\underline{B}_2 \end{pmatrix} = e_2^* \begin{pmatrix} \cos\alpha_2 & -\sin\alpha_2 \\ \sin\alpha_2 & \cos\alpha_2 \end{pmatrix} \begin{pmatrix} \hat{\underline{E}}_2 \\ \hat{c\underline{B}}_2 \end{pmatrix} \qquad \text{(III-4)}$$

that

$$dp_1/dt = e_1^* e_2^* \begin{pmatrix} 1 & \frac{1}{c}\underline{V}\times \end{pmatrix} \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} \hat{E}_2 \\ \hat{cB}_2 \end{pmatrix} \qquad \text{(V-1)}$$

where

$$\begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} = \begin{pmatrix} \cos\alpha_1 & \sin\alpha_1 \\ -\sin\alpha_1 & \cos\alpha_1 \end{pmatrix} \begin{pmatrix} \cos\alpha_2 & -\sin\alpha_2 \\ \sin\alpha_2 & \cos\alpha_2 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\alpha_1-\alpha_2) & \sin(\alpha_1-\alpha_2) \\ -\sin(\alpha_1-\alpha_2) & \cos(\alpha_1-\alpha_2) \end{pmatrix}$$

with

$$\delta = \alpha_1 - \alpha_2$$

Equation V-1 differs from the equation governing the reaction between two dual-charged particles with the same charge ratio only in that $\delta \neq 0$. But in the same manner as Eq. II-35, Eq. V-1 can be written as

$$dp_1/dt = e_1^\wedge e_2^\wedge \begin{pmatrix} \cos\delta & \sin\delta \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{c}\underline{V}_1\times \\ -\frac{1}{c}\underline{V}_1\times & 1 \end{pmatrix} \begin{pmatrix} \hat{\underline{E}}_2 \\ \hat{c\underline{B}}_2 \end{pmatrix}$$

which upon expansion yields

$$dp_1/dt = e_1^* e_2^* \left(\hat{\underline{E}}_2 + \tfrac{1}{c}\underline{V}_1 \times c\hat{\underline{B}}_2\right)\cos\delta + e_1^\wedge e_2^* \left(c\hat{\underline{B}}_2 - \tfrac{1}{c}\underline{V}_1 \times \hat{\underline{E}}_2\right)\sin\delta \qquad \text{(V-2)}$$

This equation is the most general equation governing the behavior between two dual-charged particles. In all actuality,

if one can analyze exactly the interaction between an electric charge and a magnetic monopole, then the interaction between two arbitrarily charged dual-charged particles follows.

To see this, consider Fig. V-1 which represents a phasor diagram for two dual-charged particles with different charge ratios and magnitudes.

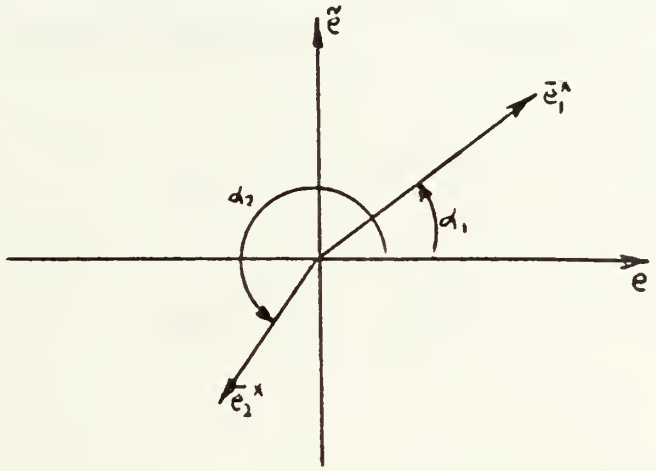

Figure V-1

Since the system is invariant under rotation in two-dimensional charge-space, an equivalent system could be represented simply by rotating the phasors $\bar{e}_1^*$ and $\bar{e}_2^*$ through the same angle $-\alpha_1$, to produce the situation shown in Fig. V-2.

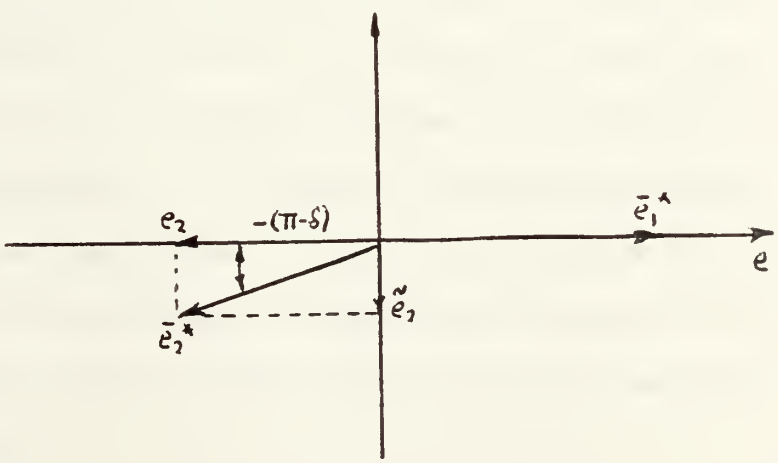

Figure V-2

In this case, the phasor e_1^* has been rotated in charge-space to an electric charge of equivalent magnitude, and hence the resultant force on e_1^* is

$$d\underline{p}_1/dt = e_1^* \left[\underline{E}_2 + \frac{1}{c} \underline{v}_1 \times c\underline{B}_2 \right] \qquad \text{(V-3)}$$

where $\underline{E}_2$ and $c\underline{B}_2$ are the actual electromagnetic field components produced by the phasor $\bar{e}_2^*$. But from Eq. III-4,

$$\underline{E}_2 = e_2 \hat{\underline{E}}_2 - e_2 c\hat{\underline{B}}_2$$

$$c\underline{B}_2 = \tilde{e}_2 \hat{\underline{E}}_2 + e_2 c\hat{\underline{B}}_2$$

where, from Fig. V-2, e_2 and $\tilde{e}_2$ are the components of the new rotated phasor $\bar{e}_2^*$. Thus Eq. V-3 becomes

$$d\underline{p}_1/dt = e_1 e_2 \left(\hat{\underline{E}}_2 + \frac{1}{c} \underline{v}_1 \times c\hat{\underline{B}}_2 \right) - e_1 \tilde{e}_2 \left(c\hat{\underline{B}}_2 - \frac{1}{c} \underline{v}_1 \times \hat{\underline{E}}_2 \right) \qquad \text{(V-4)}$$

The first term on the r.h.s. of Eq. V-4 represents the interaction between two electric charges and the second term, the interaction of an electric charge in the field produced by a magnetic monopole. In general, a solution of Eq. V-2 requires perturbation techniques. Hence, it might be of greater interest to proceed directly to the quantum mechanical treatment in the next section, where simple and well known perturbation techniques exist.

VI. THE QUANTUM MECHANICAL DESCRIPTION OF TWO INTERACTING DUAL-CHARGED PARTICLES

A. THE HYDROGENIC MODEL

As demonstrated in the previous section, a system of dual-charged particles and corresponding electromagnetic fields is invariant under rotations in two-dimensional charge-space. Therefore, if one considers a system consisting of two dual-charged particles, it should be clear that this system can be rotated in two-dimensional charge-space through a given angle α to produce an equivalent system consisting, say, of a negative electric charge and a general dual-charged particle. This procedure is illustrated in Fig. VI-1.

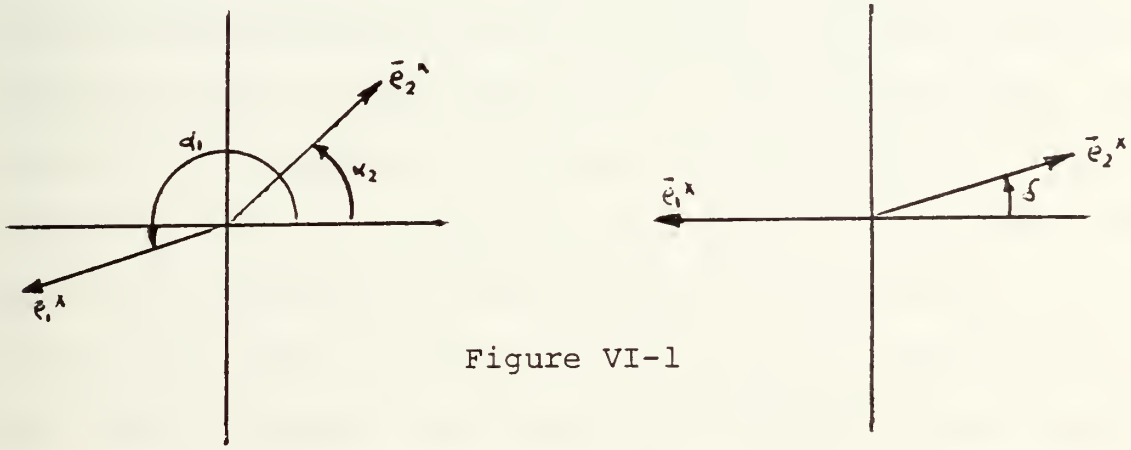

Figure VI-1

By constraining the dual-charged particle to remain at rest at the origin and assuming that the two particles have the same charge magnitude; that is $e_1^* = e_2^*$, then one has a model that is hydrogenic in nature, except that a dual-charged particle takes the place of the proton at the origin. Since, in the normal hydrogen atom, the charge ratios of the electron and

proton are related by $\alpha_1 - \alpha_2 = \pi$ (opposite charge), one can write, for the dual-charged hydrogen atom

$$\alpha_1 - \alpha_2 = \pi - \delta \qquad \text{(VI-1)}$$

Obviously, for $\delta = 0$, the system reduces to a conventional hydrogen atom. It should also be obvious that the system can also be rotated in two-dimensional charge-space to produce an equivalent system consisting of a proton at rest at the origin and an orbiting general dual-charged particle. Since the systems are equivalent, an analysis of either system is adequate to describe the general quantum mechanical interaction between two dual-charged particles with different charge ratios. In this section, the model to be analyzed shall consist of a general dual-charged particle with both electric and magnetic charge at rest at the origin and an orbiting electron. Since one will be performing a perturbation calculation, it will be necessary to place a further restriction on this model; namely, that the difference in charge ratios be given by Eq. VI-1, where δ is small. Referring to Fig. VI-1, this implies that the magnetic charge associated with the dual-charged particle at the origin is small compared to the electric charge. Thus, the magnetostatic field seen by the orbiting electron is small compared to the electric field, since in this case, one has from Eq. III-4

$$\underline{E} = \frac{e \cos\delta}{4\pi\varepsilon_0 r^2}\hat{r} \approx \frac{e}{4\pi\varepsilon_0 r^2}\hat{r} \qquad \text{(VI-2)}$$

$$c\underline{B} = \frac{e \sin\delta}{4\pi\varepsilon_0 r^2}\hat{r} \approx \frac{e\delta}{4\pi\varepsilon_0 r^2}\hat{r} \qquad \text{(VI-3)}$$

where the charge magnitude of the dual-charged particle is taken to be $e_2^* = e_1^* = e$ = magnitude of electric charge. Since it is assumed that $\delta << 1$, then $cB << E$. Although the effect of this small magnetic field on the orbit of the electron is not precisely known at this time, one might suspect that it causes a small shift in the energy levels of the hydrogen-like atom. As it turns out, this is precisely the case for some of the degenerate states of the hydrogen atom.

B. THE HAMILTONIAN FOR A DUAL-CHARGED HYDROGENIC MODEL

The force on the electron due to the electromagnetic field of the dual-charged particle at rest at the origin is given by

$$\underline{F} = -e\left[\underline{E} + \underline{v} \times \underline{B}\right]$$

where $\underline{E}$ and $\underline{B}$ are given by Eqs. VI-2 and VI-3. From any text on elementary quantum mechanics, it can easily be shown that for a charged particle in the presence of electromagnetic forces, the Hamiltonian for the electric charge, -e, is given by

$$H = \frac{(\underline{p} + e\underline{A})^2}{2m} - e\Phi \qquad \text{(VI-4)}$$

where, for this static situation, the scalar and vector potentials Φ and $\underline{A}$ are related to $\underline{E}$ and $\underline{B}$ by

$$\underline{E} = -\nabla\Phi$$

$$\underline{B} = \nabla \times \underline{A}$$

Expanding Eq. VI-4 gives

$$H = \frac{p^2}{2m} + \frac{e}{2m}\left(\underline{p}\cdot\underline{A} + \underline{A}\cdot\underline{p}\right) + \frac{e^2A^2}{2m} - e\Phi$$

or in the coordinate representation, where

$$\underline{p} = -i\hbar\nabla$$

then

$$H = -\frac{\hbar^2}{2m}\nabla^2 - e\Phi - \frac{i\hbar e}{2m}\left[A\cdot\nabla + \tfrac{1}{2}(\nabla\cdot\underline{A})\right] + \frac{e^2A^2}{2m}$$

By defining

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 - e\Phi$$

$$H' = -\frac{i\hbar e}{2m}\left[A\cdot\nabla + \tfrac{1}{2}(\nabla\cdot A)\right]$$

$$H'' = \frac{e^2}{2m}A^2$$

Eq. VI-4 becomes

$$H = H_0 + H' + H''$$

What remains now is to find the correct form of the scalar and vector potentials. By inspection, the scalar potential is obviously given by

$$\Phi = \frac{e}{4\pi\varepsilon_0 r}$$

and thus H_o, the unperturbed Hamiltonian, becomes

$$H_o = -\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{4\pi\varepsilon_o r}$$

which is the Hamiltonian associated with the hydrogen atom. The form of the vector potential $\underline{A}$ was initially introduced by Dirac [Ref. 1] as

$$\underline{A}_{\underline{n}}(\underline{r}) = \zeta \frac{\hat{r}\times\hat{n}}{r - \hat{n}\cdot\underline{r}}$$

(where ζ is some constant) and was used to describe the non-relativistic quantum mechanical interaction between an electric charge and a magnetic monopole. The curl of $\underline{A}_{\underline{n}}(\underline{r})$ describes the magnetic field of a straight solenoid of zero thickness from the origin to infinity along the direction $\hat{n}$, this solenoid being known as Dirac's string, together with a Coulomb magnetic field. The field lines of the Coulomb field join onto the field lines of the solenoid at infinity as depicted in Fig. VI-2, so that the field lines never end.

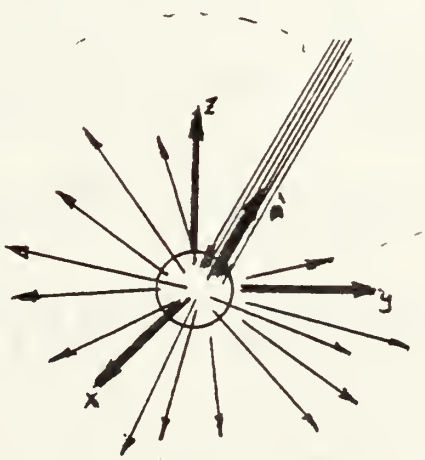

Figure VI-2

Therefore,

$$\nabla \cdot \underline{B} = \nabla \cdot (\nabla \times \underline{A}) = 0$$

Because of the string, the magnetic field determined by this vector potential does not exactly represent the Coulomb field of a magnetic monopole. However, it has since been shown [Ref. 3] that this singular string attached to the magnetic charge can be moved by a suitable gauge transformation and hence is not physically observable.

If one chooses the direction of the string to be along the positive z-axis; that is, if $\hat{n}$ is taken in the z-direction, then it can be shown that

$$\underline{A}(\underline{r}) = -\zeta \frac{(1+\cos\theta)}{r\sin\theta} \hat{\phi}$$

It should be noted that $\underline{A}$ is singular at $r = 0$ and at $\theta = 0$, where the singularity at $\theta = 0$ represents the string. To show that the curl of this vector potential does indeed give the magnetic field of Eq. VI-3 at $r \neq 0$, $\theta \neq 0$, one has

$$(\nabla \times \underline{A})_{\hat{r}} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi) - \frac{\partial}{\partial\phi}A_\theta\right] = \frac{\zeta}{r^2}$$

$$(\nabla \times \underline{A})_{\hat{\theta}} = \frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) = 0$$

$$(\nabla \times \underline{A})_{\hat{\phi}} = \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}A_r\right] = 0$$

Comparing this with Eq. VI-4, one chooses

$$\zeta = \frac{e\delta}{4\pi\varepsilon c}$$

which gives

$$\underline{A}(r) = \frac{-e\delta}{4\pi\varepsilon c} \frac{(1+\cos\theta)}{r\sin\theta} \hat{\Phi}$$

To find H' and H" for this vector potential, the components of the gradient in spherical coordinates are $\frac{\partial}{\partial r}$, $\frac{1}{r}\frac{\partial}{\partial\theta}$ $\frac{1}{r\sin\theta}\frac{\partial}{\partial\Phi}$ and hence

$$\underline{A}\cdot\nabla = \frac{-e\delta}{4\pi\varepsilon c} \frac{(1+\cos\theta)}{r^2\sin^2\theta} \frac{\partial}{\partial\Phi}$$

$$\nabla\cdot\underline{A} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\Phi}\left[\frac{-e\delta}{4\pi\varepsilon c}\frac{(1+\cos\theta)}{r\sin\theta}\right] = 0$$

Also

$$A^2 = \left[\frac{e}{4\pi\varepsilon c}\frac{1+\cos\theta}{r\sin\theta}\right]^2 \delta^2$$

Therefore

$$H' = \frac{i\hbar e^2\delta}{4\pi\varepsilon_0 mc} \frac{(1+\cos\theta)}{r^2\sin^2\theta} \frac{\partial}{\partial\Phi}$$

$$H'' = \frac{1}{2m}\left[\frac{e^2}{4\pi\varepsilon c}\frac{(1+\cos\theta)}{r\sin\theta}\right]^2\delta^2$$

Since $\delta \ll 1$, then $H' \gg H''$ and the Hamiltonian for this model is thus given, to first order in δ, by

$$H = H_0 + H' \qquad \text{(VI-5)}$$

If one lets

$m = \mu =$ reduced mass

$a_0 =$ radius of the first Bohr orbit

$$= \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2}$$

then

$$H_0 = -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r} \qquad \text{(VI-6)}$$

$$H' = \frac{i\hbar^3 \delta}{a_0 \mu^2 c} \frac{(1+\cos\theta)}{r^2 \sin^2\theta} \frac{\partial}{\partial \phi} \qquad \text{(VI-7)}$$

C. TIME-INDEPENDENT PERTURBATION THEORY

The Hamiltonian developed in the previous section for the scalar and vector potentials describing the electromagnetic field of a dual-charged particle at rest at the origin does not depend explicitly on time, and hence, time-independent perturbation theory is applicable in finding approximate solutions to the eigenvalue equation

$$H|E\rangle = E|E\rangle \qquad \text{(VI-8)}$$

where H, the Hamiltonian for the charged particle is given by Eq. VI-5. The unperturbed Hamiltonian H_o (Eq. VI-6) is the Hamiltonian of the hydrogen atom whose eigenstates and eigenvalues are exactly known and are denoted, in the customary way, by

$$H_o|n\ell m\rangle = E_n|n\ell m\rangle \qquad \text{(VI-9)}$$

The perturbing Hamiltonian H' (Eq. VI-7) is due to the magnetic field emanating from the dual-charged particle at the origin.

In time-independent perturbation theory, one assumes that a solution to Eq. VI-8 exists in the form of a convergent series; i.e.

$$|E\rangle = |b_0\rangle + \delta|b_1\rangle + \delta^2|b_2\rangle + \cdots$$

$$E = W_0 + \delta W_1 + \delta^2 W_2 + \cdots$$

To show explicitly that H' is of first order in δ, let

$$H' = \delta \tilde{H}'$$

Appropriately substituting into Eq. VI-8 gives

$$(H_0 + \delta \tilde{H}')(|b_0\rangle + \delta|b_1\rangle + \cdots) = (W_0 + \delta W_1 + \cdots)(|b_0\rangle + \delta|b_1\rangle + \cdots)$$

Since this equation is to be valid for a continuous range of sufficiently small δ, the coefficients of equal powers of δ on each side must be equal. Thus, to first order in δ

$$H_0|b_0\rangle = W_0|b_0\rangle \qquad \text{(VI-10)}$$

$$H_0|b_1\rangle + \tilde{H}'|b_0\rangle = W_0|b_1\rangle + W_1|b_0\rangle \qquad \text{(VI-11)}$$

In this case, the eigenstates of H_0 are, in general, degenerate as denoted in Eq. VI-9. Comparing this with Eq. VI-10, then

$$W_0 = E_n$$

and

$$|b_0\rangle = \sum_\alpha a_\alpha |n\alpha\rangle$$

where α stands for the parameters l,m and the a_α's are numbers,

there being s terms in the sum where one supposes that the number of different states having energy E_n is s. For this model, $s = n^2$. Normalizing $|b_o\rangle$ gives the constraint

$$\sum_{\alpha} a_\alpha^* a_\alpha = 1 \qquad \text{(VI-12)}$$

To obtain a first order correction in δ, substituting for W_o and $|b_o\rangle$ into Eq. VI-11 ultimately gives

$$|E\rangle = \sum_{\alpha} (a_\alpha + \delta \langle n\alpha | b_1 \rangle) |n\alpha\rangle + \sum_{\substack{n' \neq n \\ \alpha, \alpha'}} \frac{a_\alpha \langle n'\alpha' | H' | n\alpha \rangle}{E_n - E_{n'}} |n'\alpha'\rangle \qquad \text{(VI-13)}$$

$$E = E_n + \delta W_1 \qquad \text{(VI-14)}$$

where

$$a_{\alpha'} W_1 = \sum_{\alpha} a_\alpha \langle n\alpha' | \tilde{H}' | n\alpha \rangle \qquad \text{(VI-15)}$$

$$\langle n'\alpha' | b_1 \rangle = \sum \frac{a_\alpha \langle n'\alpha' | \tilde{H}' | n\alpha \rangle}{E_n - E_{n'}} \qquad (n' \neq n)$$

The undetermined factors $\langle n'\alpha' | \ b_1 \rangle$ for $n' = n$ are determined by orthonormalization of the states $|E\rangle$. Equation VI-15 is a system of s coupled, linear algebraic equations for the s unknowns a_α and may be written as

$$\sum_{\alpha} \left[\langle n\alpha' | \tilde{H}' | n\alpha \rangle - \delta_{\alpha'\alpha} W_1 \right] a_\alpha = 0 \qquad \text{(VI-16)}$$

This system of equations has a non-trivial solution only if

$$\det \left[\langle n\alpha' | \tilde{H}' | n\alpha \rangle - \delta_{\alpha'\alpha} W_1 \right] = 0 \qquad \text{(VI-17)}$$

Therefore, from Eqs. VI-12, VI-16 and VI-17, one can solve for the various a_q's and W_1 necessary to complete Eqs. VI-13 and VI-14.

D. ENERGY LEVEL DIAGRAMS FOR THE DUAL-CHARGED HYDROGENIC MODEL

The effect of a small magnetic charge at the origin of a hydrogen atom is to create a perturbation which should affect the normal energy levels. To first order in δ, this effect is given by Eq. VI-14. Inherent in the calculations of the energy shifts is the determination of the matrix elements, $\langle n' \; l' \; m' \mid H' \mid nlm \rangle$, given by

$$\langle n'l'm' \mid H' \mid nlm \rangle = \int u^*_{n'l'm'} \frac{i\hbar^3 \delta}{a_0 \mu^2 c} \frac{(1+\cos\theta)}{r^2 \sin^2\theta} \frac{\partial}{\partial \phi} u_{nlm} \, d^3r$$

$$= \frac{i\hbar^3 \delta}{a_0 \mu^2 c} \iiint \frac{(1+\cos\theta)}{r^2 \sin^2\theta} u^*_{n'l'm'} \frac{\partial}{\partial \phi} u_{nlm} \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{i\hbar^3 \delta}{a_0 \mu^2 c} \iiint \frac{(1+\cos\theta)}{\sin\theta} u^*_{n'l'm'} \frac{\partial}{\partial \phi} u_{nlm} \, dr \, d\theta \, d\phi \qquad \text{(VI-18)}$$

The first few hydrogen atom wave functions are

$$u_{100}(\underline{r}) = \left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-r/a_0}$$

$$u_{200}(\underline{r}) = \frac{1}{4\sqrt{2}} \left(\frac{1}{\pi a_0^3}\right)^{1/2} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$$

$$u_{210}(\underline{r}) = \frac{1}{4\sqrt{2}} \left(\frac{1}{\pi a_0^3}\right)^{1/2} \left(\frac{r}{a_0}\right) e^{-r/2a_0} \cos\theta$$

$$u_{21\pm1}(\underline{r}) = \frac{1}{8}\left(\frac{1}{\pi a_0^3}\right)^{1/2}\left(\frac{r}{a_0}\right) e^{-r/2a_0} \sin\theta\, e^{\pm i\varphi}$$

$$u_{300}(\underline{r}) = \frac{1}{9\sqrt{3}}\left(\frac{1}{\pi a_0^3}\right)^{1/2}\left(\frac{2r^2}{9a_0^2} - \frac{2r}{a_0} + 3\right) e^{-r/3a_0}$$

$$u_{310}(\underline{r}) = \frac{\sqrt{2}}{27}\left(\frac{1}{\pi a_0^3}\right)^{1/2}\left(\frac{r}{a_0}\right)\left(2 - \frac{r}{3a_0}\right) e^{-r/3a_0}\cos\theta$$

$$u_{31\pm1}(\underline{r}) = \frac{1}{27}\left(\frac{1}{\pi a_0^3}\right)^{1/2}\left(\frac{r}{a_0}\right)\left(2 - \frac{r}{3a_0}\right) e^{-r/3a_0}\sin\theta\, e^{\pm i\varphi}$$

$$u_{320}(\underline{r}) = \frac{1}{81\sqrt{6}}\left(\frac{1}{\pi a_0^3}\right)^{1/2}\left(\frac{r^2}{a_0^2}\right) e^{-r/3a_0}\left(3\cos^2\theta - 1\right)$$

$$u_{32\pm1}(\underline{r}) = \frac{1}{81}\left(\frac{1}{\pi a_0^3}\right)^{1/2}\left(\frac{r^2}{a_0^2}\right) e^{-r/3a_0}\cos\theta\sin\theta\, e^{\pm i\varphi}$$

$$u_{32\pm2}(\underline{r}) = \frac{1}{162}\left(\frac{1}{\pi a_0^3}\right)^{1/2}\left(\frac{r^2}{a_0^2}\right) e^{-r/3a_0}\sin^2\theta\, e^{\pm 2i\varphi}$$

One sees immediately that

$$\frac{\partial}{\partial\varphi} u_{nlm} = mi\, u_{nlm} \qquad \text{(VI-19)}$$

and since

$$\int_0^{2\pi} e^{i(m-m')\varphi}\, d\varphi = 0$$

if $m \neq m'$, then one can conclude that

$$\langle n'\ell'm'|H'|n\ell m\rangle = 0$$

if m = o or m ≠ m'. Furthermore, from Eqs. VI-14 and VI-17, one also notes that the calculation of the first order energy shifts involves only matrix elements between states of the same principal quantum number n. Hence, the only non-zero matrix elements of interest in the subsequent calculations will be of the form $\langle nl'\ m|H'|nlm\rangle$ where m ≠ o. Finally in view of Eq. VI-19, one also has

$$\langle n\ell'm|H'|n\ell m\rangle = -\langle n\ell'-m|H'|n\ell-m\rangle$$

where n and m are the same in both matrix elements. Therefore, the set of integrals represented by Eq. VI-18 reduces to the following set (for n≤3) in which the partial derivative and integration with respect to ϕ have been performed:

$$\langle 211|H'|211\rangle = \frac{-K}{2^5 a_o^3}\iint (1+\cos\theta)\sin\theta\, r^2 e^{-r/a_o}\, dr\,d\theta$$

$$\langle 311|H'|311\rangle = -\frac{2K}{3^6 a_o^3}\iint (1+\cos\theta)\sin\theta\, r^3\left(2-\frac{r}{3a_o}\right)^2 e^{-2r/3a_o}\, dr\,d\theta$$

$$\langle 321|H'|321\rangle = -\frac{2K}{3^5 a_o^5}\iint (1+\cos\theta)\cos^2\theta\sin\theta\, r^4 e^{-2r/3a_o}\, dr\,d\theta$$

$$\langle 322|H'|322\rangle = -\frac{K}{3^5 a_o^5}\iint (1+\cos\theta)\sin^3\theta\, r^4 e^{-2r/3a_o}\, dr\,d\theta$$

$$\langle 311|H'|321\rangle = \frac{-2K}{3^7 a_0^4}\iint(1+\cos\theta)\cos\theta\sin\theta\, r^3\left(2-\frac{r}{3a_0}\right)e^{-2r/3a_0}\,dr\,d\theta$$

where

$$K = \frac{\hbar^3 \zeta}{a_0^3 \mu^2 c}$$

These integrals are easy to perform by noting that

$$\int_0^\infty r^n e^{-ar}\,dr = \frac{n!}{a^{n+1}}$$

and therefore one obtains

$$\langle 211|H'|211\rangle = -\frac{K}{2^3} \qquad \text{(VI-20)}$$

$$\langle 311|H'|311\rangle = -\frac{K}{3^3} \qquad \text{(VI-21)}$$

$$\langle 321|H'|321\rangle = -\frac{K}{3^3} \qquad \text{(VI-22)}$$

$$\langle 322|H'|322\rangle = -\frac{2K}{3^3} \qquad \text{(VI-23)}$$

$$\langle 311|H'|321\rangle = 0 \qquad \text{(VI-24)}$$

All other matrix elements between states with $n \leq 3$ vanish.

Now in the case of the ground state, which is not degenerate, Eqs. VI-17 and VI-14 give immediately

$$E = E_1 + \langle 100 | H' | 100 \rangle = E_1$$

since $\langle 100 | H' | 100 \rangle = 0$. Hence, there is no change to the ground state energy for small δ. For the n = 2 and n = 3 states, one notes from Eqs. VI-20 to VI-24 that the matrix of the perturbing Hamiltonian H' is diagonal. Hence, the solution of Eq. VI-17 is trivial; the matrix elements of H' are the energy shifts δW_1. Thus, the energy level diagram for the n = 1, 2 and 3 states of the dual-charged hydrogen-like atom is, to first order in δ, as shown in Fig. VI-3. From this diagram, one can conclude that for $\delta \neq 0$, some of the degeneracy within the hydrogen atom energy states is removed. States with different values of m have different energies.

To obtain an appreciation for the magnitude of the shift in energy levels for small δ one notes that, at least for n = 1, 2, 3,

$$\delta W_1 = -\frac{m}{n^3} K$$

Therefore, taking the ratio of δW_1 to E_n gives

$$\left| \frac{\delta W_1}{E_n} \right| = \frac{\left(\frac{m\hbar^3}{n^3 a_0^3 \mu^2 c} \right) \delta}{\left(\frac{\hbar^2}{2 a_0^2 \mu n^2} \right)}$$

$$= \frac{m}{n} \left(\frac{2\hbar}{a_0 \mu c} \right) \delta$$

However, taking μ to be the electron mass,

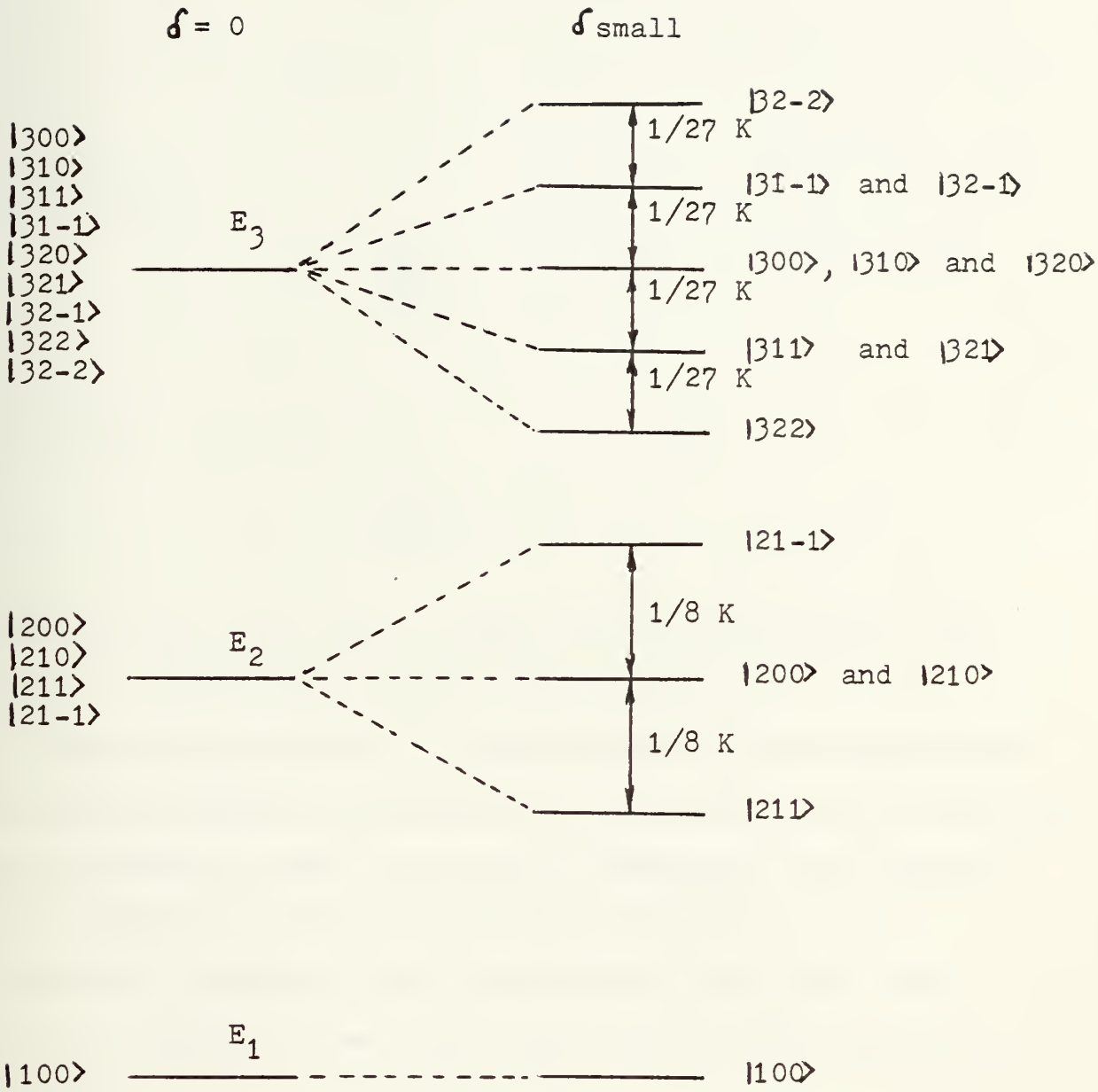
δ = 0
δ small
|300>
|310>
|311>
|31-1>
|320>
|321>
|32-1>
|322>
|32-2>
E3
|32-2>
1/27 K
|31-1> and |32-1>
1/27 K
|300>, |310> and |320>
1/27 K
|311> and |321>
1/27 K
|322>
|200>
|210>
|211>
|21-1>
E2
|21-1>
1/8 K
|200> and |210>
1/8 K
|211>
E1
|100>
|100>

Figure VI-3.

$$\frac{2\hbar}{a_e \mu c} = \frac{(2)\left(6.58 \times 10^{-16}\ \text{eV-s}\right)\left(3 \times 10^{8}\ \text{m/s}\right)}{\left(.529 \times 10^{-10}\ \text{m}\right)\left(.511 \times 10^{6}\ \text{eV}\right)}$$

$$= .015$$

and therefore

$$\left|\frac{\delta W_1}{E_n}\right| = .015\ \frac{m}{n}\ \delta$$

For $\delta \sim 10^{-1}$, one sees that

$$\left|\frac{\delta W_1}{E_n}\right| \sim 10^{-3}$$

This is about two orders of magnitude larger than the fine structure splitting caused by electron spin.

The partial removal of the degeneracy of the eigenstates of the dual-charged hydrogen-like atom results in the splitting of its spectral lines into several components. For example, where previously there was one line for the n = 2 to n = 1 transition (neglecting the fine structure and other small effects), there are now three lines, two of them spaced an interval

$$\Delta\nu = \frac{K}{8\hbar}$$

on either side of the original line.

VII. CONCLUSIONS

It has been shown that imposing a constraint of mathematical symmetry on Maxwell's equations led to the existence of magnetic monopoles coexisting with electric charges. By further hypothesizing the existence of dual-charged particles with both electric and magnetic charge, it was shown that the introduction of a two-dimensional charge-space resulted in a set of dynamical equations governing the behavior of the system that is invariant under rotation in charge-space. This invariance manifests itself in that systems in which all particles have the same magnetic to electric charge ratios are indistinguishable from conventional electrodynamic systems consisting of ordinary electric charges and electromagnetic fields. However, systems consisting of particles with different charge ratios behaved differently. It was shown that for a hydrogen-like atom consisting of two dual-charged particles with different charge ratios, some of the degeneracy within the hydrogen atom energy states was removed.

LIST OF REFERENCES

1. P. A. M. Dirac, Phys. Rev. 74, 817 (1948).

2. J. Schwinger, Science 168, 3895 (1969).

3. R. A. Brandt and J. R. Primack, Phys. Rev. D-15, 1175 (1977).

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